REAL NUMBERS

Chapter 1

Exercise-1

Part - 1 (Basic Multiple Choice Questions)

1. If prime factorization of 7560 is expressible as $2^3 \times 3^p \times q \times 7$, then the values of p and q are respectively___and__

   (a) 2, 3 (b) 5, 3 (c) 3, 5 (d) 5, 2

2. The HCF of two numbers 65 and 104 is 13. If LCM of 65 and 104 is 40x, then the value of x is :

   [CBSE (Standard) 2024]

   (a) 5 (b) 13 (c) 40 (d) 8

3. If the HCF of 567 and 693 is expressible in the form $567x + 693 \times (-4)$, find x.

   (a) 4 (b) 2 (c) 5 (d) 3

4. Let p be a prime number and k be a positive integer. If p divides k^2, then which of these is DEFINITELY divisible by p? **[CBSE Competency Focused Practice Questions 2022-23]**

k/2	k	7k	k^3

   (A) only k (b) only k and 7k (c) only k, 7k and k^3 (d) All k/2, k, 7k and k^3

5. The greatest number which when divides 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively is

   (a) 575 (b) 450 (c) 750 (d) 625

6. The sum of exponents of prime factors in the prime factorisation of 196 is

   (a) 3 (b) 4 (c) 5 (d) 2

7. The LCM of three numbers 28, 44, 132 is :

   (a) 258 (b) 231 (c) 462 (d) 924

8. If the product of two co-prime numbers is 553, then their HCF is :

   (a) 1 (b) 553 (c) 7 (d) 79

9. I. The LCM of x and 18 is 36.

   II. The HCF of x and 18 is 2.

   What is the number x?

   (a) 1 (b) 2 (c) 3 (d) 4

10. Shweta wants to organise a party. She has 336 guavas and 54 oranges at home and decided to distribute them equally among all. What is the maximum number of guests she can invite?

    (a) 5 (b) 6 (c) 7 (d) 8

Part - 2 (Basic Subjective Type Questions)

1. Show that 6^n can not end with digit 0 for any natural number 'n'.
2. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.
3. P = 2(4)(6).....(20) and Q = 1(3)(5).....(19). What is the HCF of P and Q.
4. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then n is equal to
5. The HCF of two natural numbers is 3 and their LCM is 60. The difference of two numbers is 24. Find the numbers.
6. Vicky went to a stationery shop to buy 45 pencils and 105 pens as return gifts for his birthday party. He asked the shopkeeper to pack them in such a way that each packet contains the same number of pencils or pens, with no remainder. What is the greatest number of such packets that can be made for each type? Also, how many pencils and pens will be in each packet?
7. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of the trees (only one type of tree in one row). Find the minimum number of rows required. **[CBSE Practice Set-2, 2023]**
8. Prove that $\sqrt{n}$ is not a rational number, if n is not perfect square.

Exercise-2

Part - 1 (Advanced Multiple Choice Questions)

1. $\sqrt{n}$ is a natural number such that n > 1. Which of these can DEFINITELY be expressed as a product of primes?

[CBSE Competency Focused Practice Questions 2022-23]

(i) $\sqrt{n}$ (ii) n (iii) $\frac{\sqrt{n}}{2}$

(a) only (ii) (b) only (i) and (ii)

(c) all (i), (ii) and (iii) (d) cannot be determined without knowing n

2. In a formula racing competition, the time taken by two racing cars A and B to complete 1 round of the track is 30 minutes and p minutes respectively. If the cars meet again at the starting point for the first time after 90 minutes and the HCF (30, p) = 15, then the value of p is **[CBSE APQ (Standard) 2023-24]**

(a) 45 minutes (b) 60 minutes (c) 75 minutes (d) 180 minutes

3. Find the greatest 4 digits number which leaves remainder 12 when divided by 105, 175 and 70.

(a) 9987 (b) 9462 (c) 9998 (d) 9963

4. Find the smallest 5 digits number which leaves remainder 12 when divided by 105, 175 and 70.

(a) 10500 (b) 11562 (c) 10512 (d) 11550

5. Find the least number of square tiles required to cover the floor of 630m long and 531 m broad

(a) 9 (b) 4130 (c) 81 (d) 2360

6. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF (a, b) = p^mq^n and LCM (a, b) = p^rq^s, then (m + n) (r +s) =

(a) 15 (b) 30 (c) 35 (d) 72

7. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is :

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

8. A class of 20 boys and 15 girls is divided into n groups so that each group has x boys and y girls. Values of x, y and n respectively are

(a) 3, 4 and 8 (b) 4, 3 and 6 (c) 4, 3 and 7 (d) 7, 4 and 3

9. Product of two co-prime numbers is 117. Their LCM should be

(i) 1 (ii) 117

(iii) equal to their HCF (iv) lies between 115 to 120

(a) only (ii) and (iv) (b) only (i), (ii) and (iii) (c) only (i), (iii) and (iv) (d) all (i), (ii), (iii) and (iv)

10. Which of the following statements is / are correct?

(i) There are infinitely many positive primes.

(ii) Let 'a' be a positive integer and p be a prime number such that a^2 is divisible by p, then a is divisible by p.

(iii) Every positive integer different from 1 can be expressed as a product of non-negative power of 2 and an odd number.

(iv) If 'p' is a positive prime, then $\sqrt{p}$ is an irrational number.

(a) only (i) and (iii) (b) only (i), (ii) and (iii) (c) only (i), (iii) and (iv) (d) all (i), (ii), (iii) and (iv)

11. The HCF of k and 93 is 31, where k is a natural number. Which of these can be true for some values of k?

(i) k is a multiple of 31 (ii) k is a multiple of 93

(iii) k is an even number (iv) k is an odd number

(a) only (ii) and (iii) (b) only (i), (ii) and (iii) (c) only (i), (iii) and (iv) (d) all (i), (ii), (iii) and (iv)

12. Suraj was teaching counting from 1 to 10 to his sister who is a kindergarten student. Suddenly a question came to his mind-Are there any numbers that are divisible by all the numbers from 1 to 10 (both inclusive)?

The least number Suraj can have, is : **[Module NCERT Exampler]**

(a) 10 (b) 100 (c) 504 (d) 2520

13. While coming from the school, Virat noticed that the particulate matter $PM_{2.5}$ levels were exceptionally high as he had difficulty in breathing. On reaching home, he checked the reports and found that city's Air Quality Index was 196, which was "poor" by WHO standards.
As Virat loved to play with numbers, he decided to find the prime factors of the measured value of Air Quality Index and also to evaluate the sum of exponents of prime factors of that.

The sum of exponents of prime factors in the prime factorisation of Air Quality Index value is :

(a) 3 (b) 4 (c) 5 (d) 6

Part - 2 (Advanced Subjective Type Questions)

1. Prime factorisation of three numbers A, B and C is given below :

$A = \left(2^r \times 3^p \times 5^q\right)$

$B = \left(2^p \times 3^r \times 5^p\right)$

$C = \left(2^p \times 3^q \times 5^p\right)$

such that , $p < q < r$ and p, q, and r are natural numbers.

- The largest number that divides A, B and C without leaving a remainder is 30.
- The smallest number that leaves a remainder of 2 when divided by each of A, B and C is 5402

Find A, B and C. Show your work.

2. Shweta went to a stationery shop, during a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If she wants full packs of both and the same number of pencils and crayons, the same number of pencils and crayons, how many packets of each would she need to buy?

[Module Delhi Govt. QB 2022]

3. A, B, and C starts cycling around a circular path in the same direction at same time. Circumference of the path is 1980m. If the speed of A is 330 m/min, speed of B is 198 m/min and C is 220 m/min and they start from the same point, then after what time interval they will be together at the starting point?

4. P is the LCM of 2, 4, 6, 8, 10; Q is the LCM of 1, 3, 5, 7, 9 and L is the LCM of P and Q. Then, find the relation between L and P.

5. On the two real numbers $a = 2 + \sqrt{5}$ and $b = 3 - \sqrt{7}$, perform the following operations :

[CBSE CFPQ]

(i) Calculate the sum (a + b)

(ii) Calculate the product (ab)

(iii) Find the additive inverse of a

(iv) Rationalise 1/b

(v) Verify whether the numbers a and b are rational or irrational. Provide a valid reason for your answer

6. $\sqrt{5}$ is an irrational number. Meera was asked to prove that $\left(3+\sqrt{5}\right)$ is an irrational number.

Show below are the steps of Meera's proof :

Step 1	Let $\left(3+\sqrt{5}\right)$ be a rational number. Then $\left(3+\sqrt{5}\right)$ can be written as p/q, where p and q $(q \neq 0)$ are co-primes.
Step 2	Hence, $\sqrt{5} = \left(\frac{p}{q} - 3\right)$
Step 3	Since p and q are integers $\left(\frac{p}{q} - 3\right)$ is also integer
Step 4	Since $\left(\frac{p}{q} - 3\right)$ is an integer and every integer is a rational number, $\left(\frac{p}{q} - 3\right)$ is a rational number. It implies that $\sqrt{5}$ is a rational number.
Step 5	But this contradicts the fact that $\sqrt{5}$ is an irrational number. Hence $\left(3+\sqrt{5}\right)$ is an irrational number.

She made an error in one step due to which her subsequent steps were incorrect too.

In which step did she make that error? Justify your answer.

7. Rahul and Sameer cycle around a circular track. Rahul completes one round in 18 minutes and Sameer in 24 minutes.
If they both start together at the same point and go in the same direction,

(i) After how much time will they meet at the starting point again?

(ii) How many rounds would each have completed by then?

8. In a marriage function, three types of LED light arrangements are set to flicker at different intervals: yellow lights every 3 seconds, red lights every 4 seconds, and green lights every 5 seconds. If all three lights flicker together at a certain time, determine how long it will take for them to flicker together again. Additionally, calculate how many times they will flicker simultaneously over a period of 30 minutes.

9. A conference is being attended by 60 Army participants, 84 Navy participants, and 108 Air Force participants. The organizers want to accommodate them in rooms such that each room contains the same number of participants, and all participants in a single room belong to the same department. Find the minimum number of rooms required to accommodate all the participants according to these rules.

10. Bhargav has 455 erasers and 210 pencils that he wants to distribute into several gift groups. He intends for each group to have the exact same combination of erasers and pencils, leaving no items leftover. Calculate the greatest number of groups Bhargav can create, and explain the mathematical principle used to find this number.

Part - 3 (Advanced Miscellaneous Questions)

- Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

  (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

  (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

  (c) Assertion (A) is true but reason (R) is false.

  (d) Assertion (A) is false but reason (R) is true.

1. Shilpi wants to organise a party. She has 36 kiwis and 60 oranges at home and decided to distribute them equally among all. She decides to add 42 apples also.

   Assertion (A) : The maximum number of guests that Shilpi can invite is 6.

   Reason (R) : Maximum number of guests Shilpi can invite = LCM (36, 60, 42).

2. **Assertion (A) :** Product of HCF and LCM of THREE numbers is equal to the product of those numbers.

   Reason (R) : Product of HCF and LCM of TWO numbers is equal to the product of those numbers.

3. Radha has 30 English books and 54 Mathematics books. She wants to stack them in such a way that each stack has the same number of books on a single subject.

   Assertion (A) : The total minimum number of stacks possible in this arrangement is 14.

   Reason (R) : The number of stacks of English is 9, and the number of stacks of Mathematics is 5.

4. **Assertion (A) :** The HCF of two numbers is 6 and their product is 18144, then their LCM is 3024.

   Reason (R) : HCF × LCM = Product of two given numbers.

5. **Assertion (A) :** $\sqrt{2}$ is an irrational number.

   Reason (R) : If m is a natural number which is not a perfect square, then $\sqrt{m}$ is irrational.

6. Match the Column I with Column II and choose the correct option :

Column-I	Column-II
(A) The HCF of the smallest number of two digits and the largest multiple of 5 which is less than 40 is	(i) 5
(B) HCF of $x^3y^4z^2$ and $x^2y^3z^5$, where x, y and z are distinct prime numbers is	(ii) 4
(C) The total number of factors of a prime number is	(iii) 2
(D) The sum of exponents of prime factors in the prime factorisation of 196 is	(iv) $x^2y^3z^2$

(a) A-iv, B-ii, C-i, D-iii

(b) A-i, B-iv, C-iii, D-ii

(c) A-ii, B-iv, C-i, D-iii

(d) A-ii, B-iv, C-iii, D-i

7. Match the Column I with Column II and choose the correct option :

Column-I	Column-II
(A) The HCF of 12, 21 and 15 is	(i) $3^2 \times 5^2$
(B) HCF of 112 and 198 is	(ii) 3
(C) 225 can be expressed as	(iii) 2
(D) 196 can be expressed as	(iv) $2^2 \times 7^2$

(a) A-iv, B-ii, C-i, D-iii

(b) A-i, B-iv, C-iii, D-ii

(c) A-ii, B-iv, C-i, D-iii

(d) A-ii, B-iii, C-i, D-iv

Exercise-3

(Case Based Questions)

1. LCM of several fractions $= \dfrac{\text{LCM of their numerators}}{\text{HCF of their denominators}}$

HCF of several fractions $= \dfrac{\text{HCF of their numerators}}{\text{LCM of their denominators}}$

(i) The LCM of the fractions $\frac{5}{6}, \frac{15}{24}$ and $\frac{25}{8}$ is

(a) $\frac{5}{48}$ (b) $\frac{5}{8}$ (c) $\frac{75}{48}$ (d) $\frac{75}{8}$

(ii) The HCF of $\frac{2}{5}, \frac{6}{25}$ and $\frac{8}{35}$ is

(a) $\frac{2}{5}$ (b) $\frac{24}{5}$ (c) $\frac{2}{175}$ (d) $\frac{24}{175}$

(iii) The HCF of the fractions $\frac{8}{21}, \frac{12}{35}$ and $\frac{32}{7}$ is

(a) $\frac{4}{105}$ (b) $\frac{192}{7}$ (c) $\frac{3}{105}$ (d) $\frac{5}{109}$

2. Raman is hosting the new year party for his friends. He wants to purchase some eatables like patties and buns for making burgers, some towels and some shining papers and glitter papers for decorating his house.
When he reaches the store, he finds that patties come in a pack of 6, and buns in a pack of 8. Moreover, no small towels are available and the only size available is 16 m × 20 m. The shiny paper comes in lengths of 36 inches and the glitter paper in lengths of 40 inches.

(i) Find the prime factorisation of numerical value of area of towel.

(ii) Assuming one patty is required for one bun, what is the smallest number of packs of each, Raman must buy so that no patty or bun is left out?

(iii) If a square towels has to be cut from the piece of the small towel, then find the minimum number of towels that can be cut so that there is no wastage.

3. A seminar is being conducted by an Educational Organization, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 65, 91 and 117 respectively.

(i) In each room, the same number of participants are to be seated and all of them being in the same subject, hence find the maximum number of participants that can be accommodated in each room.

(ii) What is the minimum number of rooms required during the event?

OR

(ii) The LCM of 65, 91 and 117 is

(iii) The product of HCF and LCM of 65, 91 and 117 is

(A) 692055 (B) 35360 (C) 45500 (D) 53235

4. For the screening of an informational documentary, three schools were selected by the district administration.

[CBSE CFPQ]

Name of the school	No. of students
C.A.V. Public School	78
Bal Vidya Bhawan	117
Bombay Public School	130

- During the screening, multiple rooms are used simultaneously, and each room can accommodate an equal number of students.
- All students in a particular room belong to the same school.
- As a token of appreciation, the district administration has provided an equal number of chocolates to each school.
- When distributing these chocolates, each school distributes chocolates equally among its students, ensuring fairness and consistency.

(i) Find the maximum number of students that can be seated in one room. Show your work.

(ii) What is the minimum number of rooms required? Show your work.

(iii) What is the minimum number of chocolates provided to each school? Show your work.

Answer Key

Exercise-1

Part - 1 (Basic Multiple Choice Questions)

(1) C (2) B (3) C (4) C (5) D (6) B

(7) D (8) A (9) D (10) B

Part - 2 (Basic Subjective Type Questions)

(1) There is no natural "n" for which 6^n ends with digit zero.

(2) Required Number of rooms = 17

(3) The HCF of P and Q is $3^4(5^2)$ (7)

(4) Value of n = 2

(5) The number are 6 and 30

(6) Greatest No. of packets = 15. Each packet has 3 pencils and 7 pens

(7) 12

(8) $\sqrt{n}$ is an irrational number.

Exercise-2

Part - 1 (Advanced Basic Multiple Choice Questions)

(1) B (2) A (3) B (4) C (5) B (6) C

(7) B (8) C (9) A (10) D (11) C (12) D

(13) B

Part - 2 (Advanced Subjective Type Questions)

(1) A = 600, B = 270, C = 180

(2) She needs to buy 3 packs of crayons and 4 packs of pencils

(3) 90 min

(4) L = 3(8)(5) 21 = 21P

(5) (i) $5+\left(\sqrt{5}-\sqrt{7}\right)$ (ii) $6+3\sqrt{5}-\sqrt{35}$ (iii) $-2-\sqrt{5}$

(iv) $\frac{3+\sqrt{7}}{2}$ (v) irrational

(6) Step - 3 (The error step)

(7) (i) 72 (ii) 4 (Rahul), 3 (Sameer)

(8) 60 seconds (to flicker together) ; 30 times (in 30 minutes)

(9) 21 rooms

(10) 35 groups (13 erasers + 6 pencils, using HCF)

Part - 3 (Advanced Miscellaneous Questions)

(1) C (2) D (3) C (4) A (5) A (6) B

(7) D

Exercise-3

(Case Based Questions)

(1) (i) D (ii) C (iii) A

(2) (i) Area = $16 \times 20 = 320$, Prime Factorisation = $2^6 \times 5$

(ii) LCM (6, 8) = 24, Packs $\rightarrow$ Patties = 4, Buns = 3

(iii) Maximum square size = HCF (16, 20) = 4, Number of squares = 20

(3) (i) 13 participants (ii) 21 rooms **OR** (ii) 4095 (iii) D

(4) (i) 13 students (ii) 25 rooms (iii) 1170 chocolates